The Sphere

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The Sphere

Definition:

A sphere is the locus of point which moves in such a way that its distance from a fixed point is always constant. The fixed point is called the centre of the sphere and the constant distance the radius of the sphere.

Formula:

The formula for the volume of a sphere is $\frac{4}{3}\pi r^3$, where V = volume and r = radius.

The radius of a sphere is half its diameter. So, to calculate the volume of a sphere given the diameter of the sphere, you can first calculate the radius, then the volume.

Example:

The sphere is a three-dimensional shape, also called the second cousin of a circle. A sphere is round, has no edges, and is a solid shape. The playing ball , balloon, and even light bulbs are examples of sphere shape.

Properties of a Sphere:

A sphere is a perfectly-round 3D shape with one continuous curved surface; every point on the surface is exactly the same distance from the centre. All the points of its surface are equidistant (an equal distance) from its centre.

General Equation of a Sphere:

The general equation of a sphere is $(x-a)^2+(y-b)^2+(z-c)^2=r^2$, where (a, b, c) represents the center of the sphere and r represents its radius.

Note:

When the centre of the sphere at the Origin and its radius is ‘a’ the equation of the sphere is $x^2+y^2+z^2=a^2$

Problem: 1

Find the equation of the sphere with centre (-1,2,-3) and radius 3 units.

Solution:

The equation of the sphere is

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2$$

Given Centre = (a, b, c) = (-1,2,-3)

Radius = r = 3 units

$$(x+1)^2+(y-2)^2+(z+3)^2=3^2$$

$$x^2+2x+1+y^2-4y+4+z^2+6z+9=9$$

$$x^2+y^2+z^2+2x-4y+6z+5=0$$

- **The equation** $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ **always represent a sphere and to find its centre and radius**

  The equation can be written as

  $$\left(x^2 + 2ux + u^2\right) + \left(y^2 + 2vy + v^2\right) + \left(z^2 + 2wz + w^2\right) = u^2 + v^2 + w^2 - d$$

  (i.e) $(x+u)^2 + (y+v)^2 + (z+w)^2 = \sqrt{u^2 + v^2 + w^2 - d^2}$

This shows that the equation is the locus of a point such that its distance from the fixed point (-u,-v,-w) is constant being equal to $\sqrt{u^2 + v^2 + w^2 - d^2}$.

Hence the equation represents a sphere whose centre is (-u,-v,-w) and radius $\sqrt{u^2 + v^2 + w^2 - d^2}$

Note: 1

Since there are four independent constants u, v ,w, d in the general equation of the sphere

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0.$$

A sphere can be found to satisfy four independent constant u, v, w, d in the general geometrical conditions and no more.

Note: 2

The characteristics of the equation of a sphere are:

- It is of the second degree in x, y, z
- The coefficients of x^2, y^2, z^2 are equal
- Te terms xy, yz, zx are absent

Note: 3

The general equation of any sphere is $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

This equation can be put in the form:

$$x^2 + y^2 + z^2 + \frac{2u}{a}x + \frac{2v}{a}y + \frac{2w}{a}z + d = 0$$

The centre of this sphere is $-\frac{u}{a}, -\frac{v}{a}, -\frac{w}{a}$ and its radius is $\left\{\frac{u^2}{a^2} + \frac{v^2}{a^2} + \frac{w^2}{a^2} - \frac{d}{a}\right\}^{\frac{1}{2}}$.

Note: 4

- When $u^2 + v^2 + w^2 - d = 0$, is positive, the locus is a real sphere.
- When $u^2 + v^2 + w^2 - d = 0$, is negative the locus is an imaginary sphere.
- When $u^2 + v^2 + w^2 - d = 0$, the equation of a sphere reduces to $(x+u)^2 + (y+v)^2 + (z+w)^2 = 0$, this is called a point sphere and the only real solution of the equation x = -u, y = - v, z = - w.

Problem: 2

Find the coordinates of the centre and radios of the sphere $2x^2+2y^2+2z^2-2x+4y+2z-15=0$.

Solution:

Given equation can be written as

$$x^2+y^2+z^2-x+2y+z-\frac{15}{2}=0 \qquad \textbf{(1)}$$

The general equation of the sphere is

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0 \qquad \textbf{(2)}$$

From (1) & (2)

$$2u = -1 \Rightarrow u = -\frac{1}{2}$$

$$2v = 2 \Rightarrow v = 1$$

$$2w = 1 \Rightarrow w = \frac{1}{2}$$

$$d = -\frac{15}{2}$$

Centre c = (-u, -v, -w) = (1/2, -1, -1/2)

Radius r = $\sqrt{u^2 + v^2 + w^2 - d} = \sqrt{\frac{36}{4}} = 3$

Radius r= 3 units.

Problem: 3

Find the equation of the sphere which has its centre at the point (6, -1, 2) and touch as the plane 2x-y+2z-2=0.

Solution:

The radius of the sphere is the perpendicular distance from (6, -1, 2) to the plane 2x-y+2z-2=0.

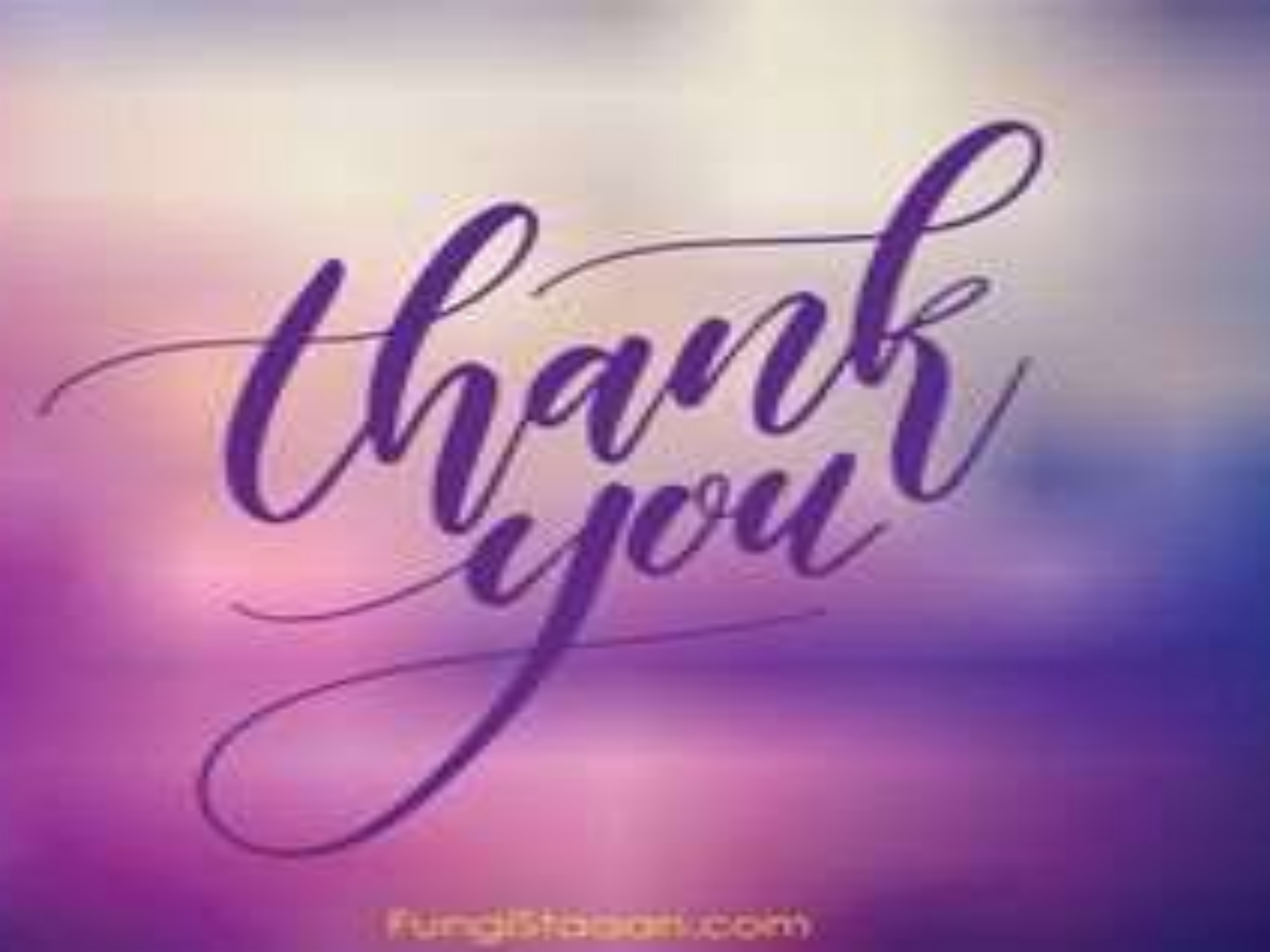
Radius of the sphere = $\frac{2(6)+1+2(2)-2}{\sqrt{2^2+1^2+2^2}} = \frac{15}{3} = 5$ units.

Hence the equation of the sphere is

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2$$

$$(x-6)^2+(y+1)^2+(z-2)^2=5^2$$

$$x^2+y^2+z^2-12x+2y-4z+16=0$$


thank you